Entanglement oscillation and survival induced by non-Markovian decoherence dynamics of entangled squeezed-state

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We study the exact decoherence dynamics of the entangled squeezed state of two single-mode optical fields interacting with two independent and uncorrelated environments. We analyze in detail the non-Markovian effects on the entanglement evolution of the initially entangled squeezed state for different environmental correlation time scales. We find that the environments have dual actions on the system: backaction and dissipation. In particular, when the environmental correlation time scale is comparable to the time scale for significant change in the system, the backaction would counteract the dissipative effect. Interestingly, this results in the survival of some residual entanglement in the final steady state.

I. INTRODUCTION

Studies on the decoherence dynamics of open quantum systems are of great importance to the field of quantum information science [1]. Any realistic analysis of quantum information protocols should take into account the decoherence effect of the environment. In many quantum communication and computation schemes, information is transmitted using photons. For instance, the first experimental verification of quantum teleportation [2] used pairs of polarization entangled photons to transfer the polarization state of one photon onto another [3]. Within a year, unconditional quantum teleportation of optical fields was demonstrated experimentally using squeezed-state entanglement [4, 5]. Given their central role in these schemes and many others, much work has been carried out on the decoherence dynamics of optical fields. In particular, several authors have studied the continuous variable entanglement of optical fields (see, for instance, Refs. [6, 7, 8, 9, 10, 11, 12]).

Conventional approaches not only treat the interactions between the quantum system S of interest and its environment E perturbatively, they also assume that the environmental correlation time τ_E is small compared to the time scale τ_0 for significant change in S. These yield approximate equations of motion, i.e. master equations, under the Born-Markov approximation [13, 14]. Indeed, many studies on the entanglement dynamics of continuous variable system relied on this approximation [6, 7, 8]. However, it is evident from recent experiments (see, for instance, Refs. [15, 16, 17]), that there are many physically relevant situations where the Markovian assumption does not hold, and a non-Markovian treatment of

the open system dynamics is necessary. So, there has been an increasing interest in the understanding of the decoherence effect of open quantum system going beyond the Born-Markovian approximation in the last decades [14, 18].

Very recently, some phenomenological models on non-Markovian entanglement dynamics of optical fields have been investigated [9, 10, 11]. It was found that in contrast to the monotonic decrease of entanglement over time in Born-Markovian entanglement dynamics [6, 7, 8], there are transient entanglement oscillations in non-Markovian ones. These oscillations are caused by the backactions of the environments on their respective local quantum systems [10, 11]. The backaction, characteristic of non-Markovian dynamics, means that the environments with their states changed due to interactions with the systems, in turn, exert their dynamical influences back on the systems.

In this paper we consider the exact decoherence dynamics of the continuous variable entangled squeezed state of two single-mode optical fields, S_1 and S_2, that are spatially separated. Each optical field, S_k, interacts with its own environment E_k ($k = 1, 2$). E_1 and E_2 are independent and uncorrelated. We study the exact entanglement dynamics of the two optical-field system for different τ_E's in comparsion with τ_0, and analyze when the system dynamics will exhibit novel non-Markovian effects, and provide a detailed description of these. To this end, we use the influence functional formalism [19, 20], developed explicitly in Refs. [12, 21, 22]. Our results show that besides the short-time oscillations, the non-Markovian effect can affect the long-time behavior of the system dynamics and the steady state as well. In particular, when τ_E is comparable to τ_0, we find that the backaction effects counteract the dissipative effects of $E_1 E_2$ on $S_1 S_2$ respectively. This leads to there being some nonzero residual entanglement in the steady state.

Our paper is organized as follows. In Sec. II, we intro-

duce a model of two single-mode optical fields in two independent and uncorrelated environments; and outline the exact dynamics that was derived in detail in Ref. [22]. In Sec. III, using logarithmic negativity as an entanglement measure of continuous-variable states, we discuss the entanglement dynamics of the entangled squeezed state. Sec. IV presents the numerical results of the entanglement dynamics, where we analyze explicitly the non-Markovian effect of the environments on the system for different τ_E's in comparison with τ_0. Finally, we conclude in Sec. V.

II. THE TOTAL HAMILTONIAN AND EXACT REDUCED SYSTEM DECOHERENCE DYNAMICS

The total Hamiltonian of the system $S_1 S_2$ plus environment $E_1 E_2$ is given by

$$H = H_{\mathrm{S}} + H_{\mathrm{E}} + H_{\mathrm{I}}, \tag{1}$$

where

$$
\begin{aligned}
H_{\mathrm{S}} &= \sum_{k=1}^{2} \hbar \omega_k a_k^\dagger a_k, \\
H_{\mathrm{E}} &= \sum_{k=1}^{2} \sum_{l} \hbar \omega_{kl} b_{kl}^\dagger b_{kl}, \\
H_{\mathrm{I}} &= \sum_{k=1}^{2} \sum_{l} \hbar (g_{kl} a_k^\dagger b_{kl} + g_{kl}^* a_k b_{kl}^\dagger), \tag{2}
\end{aligned}
$$

are, respectively, the Hamiltonian of the two optical fields, the two independent environments, and the interactions between them. The operators a_k and $a_k^\dagger$ ($k = 1, 2$) are respectively the annihilation and creation operators of the k-th optical mode with frequency ω_k. The two independent environments are modeled, as usual, by two sets of harmonic oscillators described by the annihilation and creation operators b_{kl} and $b_{kl}^\dagger$. The coupling constants between the k-th optical field and its environment are given by g_{kl}. Currently, most quantum optical experiments are performed at low temperatures and under vacuum condition. In this case, vacuum fluctuations are the main source of decoherence. Therefore, we take the environments to be at zero temperature throughout this paper.

Since we are only interested in the dynamics of $S_1 S_2$, we like to eliminate the degrees of freedom of $E_1 E_2$. The influence-functional theory of Feynman and Vernon [19] enables us to do that exactly. By expressing the forward and backward evolution operators of the density matrix of the system $S_1 S_2$ plus environment $E_1 E_2$ as a double path integral in the coherent-state representation [23], and performing the integration over the degrees of freedom of $E_1 E_2$, we incorporate all the environmental effects on $S_1 S_2$ in a functional integral named influence functional [12, 19, 22]. The reduced density matrix,

which fully describes the dynamics of $S_1 S_2$ is given by

$$
\begin{aligned}
\rho(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t) = \int d\mu(\boldsymbol{\alpha}_i) d\mu(\boldsymbol{\alpha}'_i) \mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t | \bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) \\
\times \rho(\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0), \tag{3}
\end{aligned}
$$

where $\rho(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t) = \langle \boldsymbol{\alpha}_f | \rho(t) | \boldsymbol{\alpha}'_f \rangle$ is the reduced density matrix expressed in coherent-state representation and $\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t | \bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0)$ is the propagating function. In the derivation of Eq. (3), we have used the coherent-state representation

$$
|\boldsymbol{\alpha}\rangle = \prod_{k=1}^{2} |\alpha_k\rangle, \ |\alpha_k\rangle = \exp(\alpha_k a_k^\dagger)|0_k\rangle. \tag{4}
$$

which are the eigenstates of annihilation operators, i.e. $a_k |\alpha_k\rangle = \alpha_k |\alpha_k\rangle$ and obey the resolution of identity, $\int d\mu(\boldsymbol{\alpha}) |\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}| = 1$ with the integration measures defined as $d\mu(\boldsymbol{\alpha}) = \prod_l e^{-\bar{\alpha}_l \alpha_l} \frac{d\bar{\alpha}_l d\alpha_l}{2\pi i}$. $\bar{\boldsymbol{\alpha}}$ denotes the complex conjugate of $\boldsymbol{\alpha}$.

The time evolution of the reduced density matrix is determined by the propagating function $\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t | \bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0)$. The propagating function is expressed as the path integral governed by an effective action which consists of the free actions of the forward and backward propagators of the optical-field system and the influence functional obtained from the integration of environmental degrees of freedom. After evaluation of the path integral, the final form of the propagating function is obtained as follows

$$
\begin{aligned}
\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t | \bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \exp \Big\{ \sum_{k=1}^{2} \big[u_k(t) \bar{\alpha}_{kf} \alpha_{ki} \\
+ \bar{u}_k(t) \bar{\alpha}'_{ki} \alpha'_{kf} + [1 - |u_k(t)|^2] \bar{\alpha}'_{ki} \alpha_{ki} \big] \Big\}, \tag{5}
\end{aligned}
$$

where $u_k(\tau)$ satisfies

$$
\dot{u}_k(\tau) + i\omega_k u_k(\tau) + \int_0^\tau \mu_k(\tau - \tau') u_k(\tau') = 0 \tag{6}
$$

with $\mu_k(x) \equiv \sum_l e^{-i\omega_l x} |g_{kl}|^2$ being used. Combining Eq. (5), we can get the exact time-dependent state from any initial state by the evaluation of the integration in Eq. (3).

To compare with the conventional master equation description of such system, we now derive a master equation from the above results. After taking the time derivative to Eq. (3) and recalling the explicit form of Eq. (5), we can derive an exact master equation

$$
\begin{aligned}
\dot{\rho}(t) = -\frac{i}{\hbar} [H'(t), \rho(t)] + \sum_{k=1}^{2} \Gamma_k(t) [2a_k \rho(t) a_k^\dagger \\
- a_k^\dagger a_k \rho(t) - \rho(t) a_k^\dagger a_k], \tag{7}
\end{aligned}
$$

where

$$
H'(t) = \sum_{k=1}^{2} \hbar \Omega_k(t) a_k^\dagger a_k, \tag{8}
$$

is the modified Hamiltonian of the two optical modes and

$$\frac{\dot{u}_k(t)}{u_k(t)} \equiv -\Gamma_k(t) - i\Omega_k(t). \tag{9}$$

Eq. (7) is the exact master equation for the optical-field system. $\Omega_k(t)$ plays the role of a time-dependent shifted frequency of the k-th optical field. $\Gamma_k(t)$ represents the corresponding time-dependent decay rate of the field. We emphasize that the derivation of the master equation goes beyond the Born-Markovian approximation and contains all the backactions between the system and the environments self-consistently. All the non-Markovian character resides in the time-dependent coefficients of the exact master equation.

The time-dependent coefficients in the exact master equation, determined by Eq. (9), essentially depend on the so-called spectral density, which characterizes the coupling strength of the environment to the system with respect to the frequencies of the environment. It is defined as $J_l(\omega) = \sum_k |g_{lk}|^2 \delta(\omega - \omega_l)$. In the continuum limit the spectral density may have the form

$$J_k(\omega) = \eta_k \omega \left(\frac{\omega}{\omega_c}\right)^{n-1} e^{-\frac{\omega}{\omega_c}}, \tag{10}$$

where ω_c is an exponential cutoff frequency, and η_k is a dimensionless coupling constant between S_k and E_k. The environment is classified as Ohmic if $n = 1$, sub-Ohmic if $0 < n < 1$, and super-Ohmic for $n > 1$ [20, 24]. Different spectral densities manifest different non-Markovian decoherence dynamics.

We note that our exact master equation reduces to the conventional master equation under the relevant Markov approximation. The coefficients in the master equation (7) become time-independent [22]

$$\begin{aligned}
\Gamma_k(t) &= \pi J_k(\omega_k), \\
\Omega_k(t) &= \omega_k - \mathcal{P} \int_0^{+\infty} \frac{J(\omega)d\omega}{\omega - \omega_k},
\end{aligned} \tag{11}$$

where $\mathcal{P}$ denotes the Cauchy principal value. The coefficients in Eqs. (11) are precisely the corresponding ones in the Markovian master equation of the optical system [13].

III. THE NON-MARKOVIAN ENTANGLEMENT DYNAMICS OF THE ENTANGLED SQUEEZED STATE

Initially at time $t = 0$, $S_1 S_2$ is in an entangled squeezed state. The entangled two-mode squeezed state is defined as the vacuum state acted on by the two-mode squeezing operator

$$|\psi(0)\rangle = \exp[r(a_1 a_2 - a_1^\dagger a_2^\dagger)]|00\rangle, \tag{12}$$

where r is the squeezing parameter. In the coherent-state representation, this initial state is given by

$$\rho(\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \frac{\exp[-\tanh r(\bar{\alpha}_{1i}\bar{\alpha}_{2i} + \alpha'_{1i}\alpha'_{2i})]}{\cosh^2 r}. \tag{13}$$

The state approaches the ideal Einstein-Podolsky-Rosen (EPR) state [25] in the limit of infinite squeezing ($r \to \infty$). The traditional way to generate the entangled two-mode squeezed state is via the nonlinear optical process of parametric down-conversion [26]. Recently, a microwave cavity QED-based scheme to generate such states has also been proposed [27]. After generating the entangled state given by Eq. (12), the two cavity fields are then propagated, respectively, to the two locations separated between the sender and the receiver. A quantum channel is thus established through the entangled two-mode squeezed state and is ready for teleporting unknown optical coherent states [4, 5].

At $t > 0$, due to interactions with $E_1 E_2$, $|\psi(0)\rangle$ evolves to a mixed state. A straightforward way to obtain the time-dependent mixed state is by integrating the propagating function over the initial state of Eq. (3). Then the time-evolution solution of the reduced density matrix can be obtained exactly,

$$\rho(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t) = a \exp\left[\sum_{k \neq k'} \left(\frac{b}{2}\bar{\alpha}_{kf}\bar{\alpha}_{k'f} + c\bar{\alpha}_{kf}\alpha'_{kf} + \frac{b^*}{2}\alpha'_{kf}\alpha'_{k'f}\right)\right], \tag{14}$$

where

$$a = \frac{1}{\cosh^2 |r| [1 - \tanh^2 |r| (1 - |u(t)|^2)^2]}, \tag{15}$$

$$b = \frac{-\tanh |r| u(t)^2}{1 - \tanh^2 |r| (1 - |u(t)|^2)^2}, \tag{16}$$

$$c = \frac{\tanh^2 |r| (1 - |u(t)|^2) |u(t)|^2}{1 - \tanh^2 |r| (1 - |u(t)|^2)^2}. \tag{17}$$

To measure the entanglement in continuous variable system, one generally uses the logarithmic negativity [28]. The logarithmic negativity of a bipartite system was introduced originally as

$$E_N = \log_2 \sum_i |\lambda_i^-|, \tag{18}$$

where λ_i^- is the negative eigenvalue of ρ^{T_i}, and ρ^{T_i} is a partial transpose of the bipartite state ρ with respect to the degrees of freedom of the i-th party. This measure is based on the Peres-Horodecki criterion [29, 30] that a bipartite quantum state is separable if and only if its partially transposed state is still positive.

For the continuous-variable (Gaussian-type) bipartite state, its density matrix is characterized by the covariance matrix defined as the second moments of the quadrature vector $X = (x_1, p_1, x_2, p_2)$,

$$V_{ij} = \frac{\langle \Delta X_i \Delta X_j + \Delta X_j \Delta X_i \rangle}{2}, \tag{19}$$

where $\Delta X_i = X_i - \langle X_i \rangle$, and $x_i = \frac{a_i + a_i^\dagger}{\sqrt{2}}$, $p_i = \frac{a_i - a_i^\dagger}{i\sqrt{2}}$. The canonical commutation relations take the form as $[X_i, X_j] = iU_{ij}$, with $U = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}$ and $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

defining the symplectic structure of the system. The property of the covariance matrix V is fully determined by its symplectic spectrum $\nu = (\nu_1, \nu_2)$, with $\pm\nu_i$ ($\nu_i > 0$) the eigenvalues of the matrix: iUV. The uncertainty principle exerts a constraint on ν_i such that $\nu_i \geqslant \frac{1}{2}$ [31]. Thus the Peres-Horodecki criterion for the continuous-variable state can be rephrased as the state being separable if and only if the uncertainty principle, $V + \frac{i}{2}U \geqslant 0$, is still obeyed by the covariance matrix under the partial transposition with respect to the degrees of freedom of a specific subsystem [32]. In terms of phase space, the action of partial transposition amounts to a mirror reflection with respect to one of the canonical variables of the related subsystem. For instance, $\tilde{V} = \Lambda V \Lambda$, and $\Lambda = diag(1,1,1,-1)$ is the partial transposition with respect to the second subsystem. If a Gaussian-type bipartite state is nonseparable, the covariance matrix $\tilde{V}$ will violate the uncertainty principle and its symplectic spectrum $\tilde{\nu} = (\tilde{\nu}_1, \tilde{\nu}_2)$ will fail to satisfy the constraint $\tilde{\nu}_i \geqslant \frac{1}{2}$. The logarithmic negativity is then used to quantify this violation as [28]

$$E_N = \max\{0, -\log_2(2\tilde{\nu}_{\min})\}, \qquad (20)$$

where $\tilde{\nu}_{\min}$ is the smaller one of the two symplectic eigenvalues $\tilde{\nu}_i$. It is evident from Eq. (20) that, if $\tilde{V}$ obeys the uncertainty principle, i.e., $\tilde{\nu}_i \geqslant \frac{1}{2}$, then $E_N(\rho) = 0$, namely, the state is separable. Otherwise, it is entangled. Therefore, the symplectic eigenvalue $\tilde{\nu}_{\min}$ encodes a qualitative feature of the entanglement for an arbitrary continuous-variable bipartite state.

With this entanglement measure at hand, we study now the entanglement dynamics of the squeezed-state quantum channel in our model. From the time-dependent state, the covariance matrix for the optical field can be calculated straightforwardly,

$$V = \begin{pmatrix} \frac{y(1+d)}{2(1-d)^2} & 0 & \frac{a\mathrm{Re}[b]}{x} & \frac{a\mathrm{Im}[b]}{x} \\ 0 & \frac{y(1+d)}{2(1-d)^2} & \frac{a\mathrm{Im}[b]}{x} & \frac{-a\mathrm{Re}[b]}{x} \\ \frac{a\mathrm{Re}[b]}{x} & \frac{a\mathrm{Im}[b]}{x} & \frac{y(1+d)}{2(1-d)^2} & 0 \\ \frac{a\mathrm{Im}[b]}{x} & \frac{-a\mathrm{Re}[b]}{x} & 0 & \frac{y(1+d)}{2(1-d)^2} \end{pmatrix}, \qquad (21)$$

where $x = [(1-c)^2 - |b|^2]^2$, $y = \frac{a}{1-c}$, and $d = c + \frac{|b|^2}{1-c}$. And the logarithmic negativity $E_N(t)$ can also be obtained exactly from Eq. (20). It is easy to verify that the initial entanglement is $E_N(0) = \frac{2r}{\ln 2}$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the following, we analyze explicitly the exact decoherence dynamics of the entangled squeezed state of $S_1 S_2$ under the influence of $E_1 E_2$. For simplicity, we assume from here on that the two optical fields are identical, i.e., $\omega_1 = \omega_2 \equiv \omega_0$; and they interact with the same strength, $g_{1l} = g_{2l} \equiv g_l$, with their individual environments. For
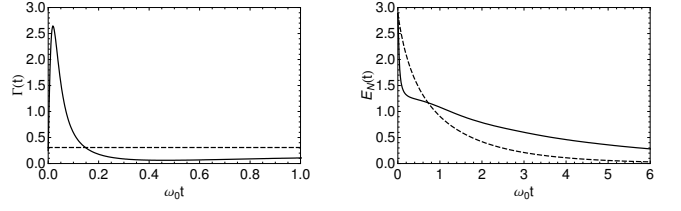

FIG. 1: Damping rate $\Gamma(t)$ and logarithmic negativity $E_N(t)$ for the entangled squeezed state as function of dimensionless quantity $\omega_0 t$ and their corresponding Markovian results (dashed lines). The parameters $\omega_c/\omega_0 = 50.0$, $\eta = 0.1$, and $r = 1.0$ are used in the numerical calculation.
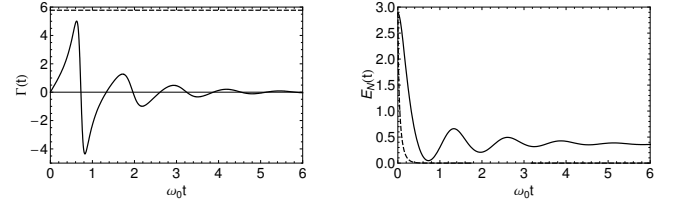

FIG. 2: Damping rate $\Gamma(t)$ and logarithmic negativity $E_N(t)$ for the entangled squeezed state as function of dimensionless quantity $\omega_0 t$ and their corresponding Markovian results (dashed lines). The parameters $\omega_c/\omega_0 = 1.0$, $\eta = 5.0$, and $r = 1.0$ are used in the numerical calculation.

definiteness, we consider both E_1 and E_2 to have Ohmic spectral density. The environmental correlation time τ_E in this case is roughly inversely proportional to the cutoff frequency ω_c in Eq. (10), i.e., $\tau_E \simeq 1/\omega_c$ [18]. It is emphasized that the cutoff frequency ω_c, which is originally introduced to eliminate infinities in frequency integrations, therefore also determines if the dynamics of open system S is Markovian or non-Markovian. Our nonperturbatively derived exact results allow us to explore all these possibilities.

In Fig. 1, we plot the numerical results of the decay rate $\Gamma(t)$ and logarithmic negativity $E_N(t)$ when $\tau_E \ll \tau_0$. The positivity of $\Gamma(t)$ throughout the whole evolution process guarantees the monotonic decrease of $E_N(t)$. Accordingly, the entangled squeezed state even-
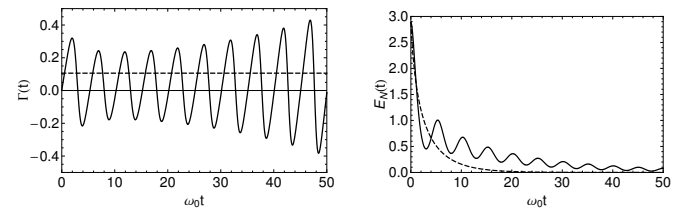

FIG. 3: Damping rate $\Gamma(t)$ and logarithmic negativity $E_N(t)$ for the entangled squeezed state as function of dimensionless quantity $\omega_0 t$ and their corresponding Markovian results (dashed lines). The parameters $\omega_c/\omega_0 = 0.2$, $\eta = 5.0$, and $r = 1.0$ are used in the numerical calculation.

tually evolves to a product state, namely the ground state of the system: $\rho_g = |00\rangle\langle 00|$. Clearly, in this case, the backactions of E_1E_2 have a negligible effect on the dynamics of S_1S_2, and we say the system dynamics is mainly governed by the dissipative effect of the environments. There is thus no qualitative difference between the exact entanglement dynamics and the Markovian results. Quantitatively, however, we note that for $t < \tau_E$, the distinctive increase of $\Gamma(t)$ results in $E_N(t)$ decreasing rapidly. This non-Markovian effect only shows up in a very short time scale. In fact, for $t > \tau_E$, $\Gamma(t)$ decreases and approaches gradually to a constant value as t approaches τ_0; and the rate of decrease of $E_N(t)$ decreases.

Fig. 2 shows $\Gamma(t)$ and $E_N(t)$ when $\tau_E = \tau_0$. In this case, the backactions of E_1E_2 have a considerable impact on the dynamics of S_1S_2, and the Markovian approximation is not applicable. Firstly, we note that $\Gamma(t)$ can take negative values. Physically, this corresponds to the systems reabsorbing photons from the environments, which leads to an increase in the photon number of the systems [14]. These negative decay rates provide evidences for backactions in non-Markovian dynamics [34]. Secondly, we observe that $\Gamma(t)$ approaches zero asymptotically. Both results clearly differ from the Markovian ones. Consequently, $E_N(t)$ presnts distinctive behaviors that are absent in the Markovian results. Firstly, due to the negative decay rates, $E_N(t)$ shows oscillations. We must emphasize that these oscillations are fundamentally different from the transient entanglement oscillations previously obtained when two optical-fields interact with a common environment [12]. They are caused by the backactions of the environments on their respective local optical fields and are characteristic of non-Markovian dynamics. Similar oscillations have been obtained in a system of two two-level atoms in two separated damping cavities [33]. Secondly, and more interestingly, we find that there is some residual entanglement left in the steady state. From previous studies [9, 10, 12], one would have concluded that non-Markovian effects only show up in short-time dynamics. Our results, however, clearly show on the contrary that non-Markovian effects can also have an influence on the long-time behavior of the system dynamics and the final steady state of the system. This counterintuitive behavior can be explained with the fact that the dissipative influence on the entanglement dynamics by the environments is strongly counteracted by the effect due to their backactions. Consequently, the decay of the entanglement ceases when the system evolves to some steady state, which is not the ground state ρ_g.

The results when $\tau_E \gg \tau_0$ are shown in Fig. 3; a situation considered in Ref. [11]. Due to extremely long memory of the environments, the backactions on the systems are so strong that they govern the decoherence dynamics. As a result, $\Gamma(t)$ and hence $E_N(t)$ oscillate over a very long duration. These oscillations persist even as the state approaches the ground state. The 'equilibrium' position for the oscillation of $\Gamma(t)$ is not at zero, but a small pos-

itive value. This positivity means the systems dynamics will experience a weak dissipation, which is verified by the time evolution of $E_N(t)$ in Fig. 3.

In summary, we have studied the exact entanglement dynamics of the two optical-field system for different τ_E's in comparison with τ_0. Specifically, we have analyzed when the system dynamics will exhibit novel non-Markovian effects, and provided a detailed description of these.

V. CONCLUSIONS

We have applied the influence-functional method of Feynman and Vernon to investigate the exact entanglement dynamics of two single-mode optical fields S_1S_2 coupled to two independent and uncorrelated environments E_1E_2. From our analytical and numerical results, it is seen that E_1E_2 exert two competing influences on our system. One effect, $\mathcal{D}$, is dissipative and is responsible for the decoherence of S_1S_2. The other, $\mathcal{B}$, is due to the backactions of E_1E_2 on S_1S_2. The degree of manifestations of $\mathcal{D}$ and $\mathcal{B}$ in the dynamics of S_1S_2 depends on τ_E in comparison with τ_0. For $\tau_E \ll \tau_0$, $\mathcal{D}$ dominates and $\mathcal{B}$ only gives rise to a transient coherent oscillation of S_1S_2. The state of S_1S_2 evolves to the ground state ρ_g, which is coincident with the Markovian result. If $\tau_E = \tau_0$, the near resonant interaction between S_1S_2 and E_1E_2 results in $\mathcal{D}$ and $\mathcal{B}$ being comparable and counteract each other. These give rise to transient negative decay rates and asymptotical zero decay rate. The state of S_1S_2 thus evolves asymptotically to some steady state, which is not the ground state ρ_g. Finally, when $\tau_E \gg \tau_0$, $\mathcal{B}$ dominates and governs the dynamics of S_1S_2. The decay rates of the system oscillate about some non-negative equilibrium position over a very long duration. This slight positivity guarantees an overall weak dissipative effect on the system dynamics. Therefore, the state of S_1S_2 eventually approaches the ground state with the entanglement oscillation persisting on for a very long time.

The theory we have established is a non-perturbative description of the exact decoherence dynamics of a system of two single-mode optical fields. It can serve as a useful basic theoretical model in analyzing the non-Markovian decoherence dynamics of optical fields employed in practical quantum information processing schemes. It should be noted that although only the Ohmic spectral density is considered here, it is straightforward to generalize our discussion to the non-Ohmic cases.

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